Transformer Derating for Harmonic Currents: A Wide-Band Measurement Approach for Energized Transformers

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Abstract—Power system transformers must often be derated when supplying harmonic currents to nonlinear loads. IEEE Recommended Practice C57.110 extrapolates transformer loss for harmonic frequencies based on dc winding resistance and rated eddy-current loss. This paper describes an improved derating technique based on direct measurements performed at fundamental and harmonic frequencies. The measurements can be performed regardless of whether the transformer is energized and in service or deenergized and out of service. For energized transformers, the measurement can be performed regardless of whether the transformer is fully loaded or unloaded. Measurements of several distribution transformers show the improvement of this method over C57.110.

Index Terms—Harmonics, impedance measurement, power quality, transformer derating.

I. INTRODUCTION

TRANSFORMERS are traditionally designed for operation with line-frequency voltage and current. However, with the increased application of power electronics, this assumption is frequently violated. Transformers must often supply harmonic currents to nonlinear power-electronic loads. Proper application of transformers in this harmonic environment requires derating to prevent overload and premature failure. This paper first reviews IEEE Recommended Practice C57.110 [1] that derates transformers using a calculation based on the dc winding resistance and the rated load loss.

As an improved alternate to the calculation of C57.110, this paper describes a derating method based on wide-band measurement of transformer effective winding resistance. The measurement uses instrumentation originally developed for measuring line impedance [2]–[4]. The measurement can be performed whether the transformer is deenergized or energized, and energized transformers can be measured in any condition between no load and full load. To illustrate and validate the derating method, a test transformer with a known winding design is measured and compared to an existing theoretical analysis and to a finite-element analysis (FEA). Measurements performed on three power system transformers with unknown winding designs are also presented. These measurements show that the assumptions of C57.110 do not universally apply to all transformers.

II. IEEE RECOMMENDED PRACTICE C57.110

IEEE Recommended Practice C57.110 describes one derating method for transformers carrying harmonic currents. This method divides the total transformer load loss $P_{LL}$ into three categories

$$P_{LL} = P + P_{EC} + P_{OSEL}.$$  (1)

Using the dc winding resistance “R” and the rms winding current “$I_h$,” the loss $P = I^2R$ might be called the “dc loss” because the calculation ignores the role of the load ac current spectrum in causing eddy-current loss. In contrast, the eddy-current loss $P_{EC}$ represents the loss caused by eddy currents in the winding and does depend on the load current spectrum. Other stray losses are lumped together in the term $P_{OSEL}$, which represents eddy-current loss in the bus bars, connections, and structural parts.

In C57.110, the eddy-current loss $P_{EC}$ is assumed to rise in proportion to the square of the harmonic number “$h$.” If $I_R$ is the line-frequency rated-load current, $P_{EC-R}$ is the rated eddy-current loss for $I_R$, $I_h$ is the rms current at each harmonic, and $h_{max}$ is the highest harmonic of interest. C57.110 gives the eddy-current loss $P_{EC}$ as

$$P_{EC} = P_{EC-R} \sum_{h=1}^{h_{max}} \left( \frac{I_h}{I_R} \right)^2 I_h^2.$$  (2)

This calculation assumes that the ratio $P_{EC-R}$ is known or measurable. According to C57.110, the
value of $P_{dc-R}$ is determined by taking the difference between the calculated dc loss $P_{dc-R}$ and the measured load loss $P_{LL}$ at line frequency. If an effective rated eddy-current resistance $R_{EC-R}$ is defined such that

$$R_{EC-R} = \frac{P_{dc-R}}{I_R}$$

(3)

an equivalent formulation of (2) is

$$P_{EC} = \sum_{h=1}^{h_{max}} I_h^2 R_{EC-R} h^2$$

(4)

where for each harmonic number $R_{EC-R}h^2$ might be called the eddy-current resistance for each harmonic.

In C57.110, the other stray loss $P_{OSL}$ is assumed to rise in proportion to the 0.8 power of the harmonic number. While $P_{OSL}$ “is generally not a consideration for dry-type transformers, it can have a substantial effect on liquid-filled transformers.” The other stray loss $P_{OSL}$ is calculated from the rated other stray loss $P_{OSL-R}$ as

$$P_{OSL} = P_{OSL-R} \sum_{h=1}^{h_{max}} \left( \frac{I_h}{I_R} \right)^2 h^{0.8}$$

(5)

Similarly to (3), an effective rated other stray loss resistance $R_{OSL-R}$ is defined as

$$R_{OSL-R} = \frac{P_{OSL-R}}{I_R}$$

(6)

and similar to (4), an equivalent formulation for other stray loss is

$$P_{OSL} = \sum_{h=1}^{h_{max}} I_h^2 R_{OSL-R} h^{0.8}$$

(7)

where for each harmonic number $R_{OSL-R}h^{0.8}$ might be called the other stray loss resistance for each harmonic.

In this equivalent and alternate formulation of C57.110, the dc winding resistance $R$, the eddy-current resistance $R_{EC-R}h^2$, and the other stray loss resistance $R_{OSL-R}h^{0.8}$ are combined into an overall effective winding resistance $R_h$ that represents the sum of all losses for a given harmonic

$$R_h = R + R_{EC-R} h^2 + R_{OSL-R} h^{0.8}$$

(8)

and (8) is used in (1) to find the load loss as

$$P_{LL} = \sum_{h=1}^{h_{max}} I_h^2 R_h \star$$

(9)

Therefore, in this equivalent formulation, C57.110 estimates the effective winding resistance $R_h$ at each harmonic, calculates the load loss $I_h^2 R_h$, for each harmonic, and adds the loss at each harmonic to find the total load loss. This paper describes a technique for measuring $R_h$ directly rather than calculating it using C57.110.

### III. Test Transformer

Fig. 1 shows the geometry of a small test transformer specifically designed for the study of transformer loss. Measurements performed on this transformer were compared to an existing theory and an FEA. The transformer design is listed in Table I and uses a primary and secondary winding on a “shell”-type cut-C core. For convenience, only one side of the transformer winding is shown in Fig. 1. Fig. 1 also shows a set of $x$-$y$-$z$ Cartesian coordinate axes used for analysis.

The test transformer was carefully constructed to justify several assumptions which are required for the success of the theoretical analysis and FEA, but are not required for the success of the measurement described in this paper. Fig. 1 also represents these assumptions. First, the winding layers, which are actually composed of many turns of round wire, are approximated as a single foil conductor which extends the length of the winding window $b_{win}$. The effective thickness “$b$” of the foil conductor is the thickness of a square conductor with the same cross-sectional area as the actual round conductor. However, when extended across the window to length $b_{win}$, the equivalent foil conductor has a greater cross-sectional area than the actual wire in the layer. A correction factor called

![Integration Primary Winding Secondary Magnetic Path](image-url)
porosity $p$ is introduced to correct for this approximation. The value of $p < 1$, is the ratio of the total cross-sectional area of all turns in the layer to the cross-sectional area of the equivalent foil conductor that models the winding layer. The test transformer was carefully wound to minimize this inaccuracy and to make the foil winding approximation as accurate as possible. Second, the permeability of the core material is assumed to be so large that the leakage magnetic field $H_y$ is everywhere parallel to the center leg of the transformer. As a result, there is no significant magnetic field in the core and the leakage field is constrained to the winding layers in the $y$ direction, as shown in Fig. 1. The current density $J_z$ that induces the magnetic field $H_y$ is constrained to the $z$ direction, as shown in Fig. 1. Finally, the windings are assumed to extend infinitely in the $z$ direction and the end-turn effects are neglected.

IV. FEA

The transformer shown in Fig. 1 was analyzed using the two-dimensional (2-D) Ansoft finite-element software “Eddy” [5]. Fig. 2 shows plots of $H_y$ and $J_z$ obtained from this analysis for frequencies at dc and 8 kHz. These plots provide physical insight into the distribution of leakage field and winding currents at dc and high frequency, as well as an estimate of winding loss. The $x$ axis in the plots of Fig. 2 corresponds to the distance along the $x$ axis of Fig. 1 starting from center-leg point of zero current and magnetic field.

For dc, the distributions of $H_y$ and $J_z$ are straightforward. The dc current density $J_z$ in Fig. 2(b) remains constant as a function of $x$, showing that the current is evenly distributed in the conductor. The corresponding dc magnetic field in Fig. 2(a) changes linearly within each layer, and it remains constant in the space between layers. This result is obtained by the application of Ampere’s law around a closed integration path linking a portion of the winding. Ampere’s law is stated as

$$ \int H \cdot dl = I_{\text{enc}} $$

where $I_{\text{enc}}$ is the current enclosed by the path $dl$. The example path shown in Fig. 1 encloses the two innermost winding layers and a portion of the magnetic core. As the core is assumed to have infinite permeability and, therefore, cannot support a magnetic field, the magnetic field $H_y$ is parallel to the winding layers and extends across the winding window for a distance $b_{\text{win}}$. If the total current in each layer is $I$, then for the example path of Fig. 1, the value of $H_y$ is simply $2I/b_{\text{win}}$. The value of $H_y$ as a function of $x$ is determined by moving the right-hand segment of the integration path. Beginning at $x = 0$ in Fig. 2(a), the value of $H_y$ climbs linearly with $x$ while within each winding layer of the primary and is constant in the spaces between the winding layers. The value of $H_y$ declines as $x$ is increased through the secondary layers as the secondary current gradually balances the primary current. As $x$ moves past the outmost secondary layer, the value of $H_y$ becomes zero as the current flowing in the secondary layers exactly balances the current flowing in the primary layers.

For higher frequencies, the eddy currents induced in the winding layers by the leakage flux change the leakage field and current distribution in the winding layers. Fortunately, the leakage field in the nonconductive space between the winding layers is governed by Ampere’s Law and does not change. This fact is shown in Fig. 2(a) by comparing $H_y$ at 8 kHz to $H_y$ at dc in the spaces between the layers. This property establishes the boundary conditions necessary for the derivation of eddy-current loss formulas in the theoretical analysis described in a subsequent section.

Another approach to understanding the current distributions is to consider the skin depth calculation for a winding layer. Fig. 2(b) shows that the peak current densities are located at the layers’ edges which correspond to the peaks in leakage field. This effect is described mathematically by the skin depth equation

$$ \delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma p}} $$

In this equation, the conductivity $\sigma$ of the winding is reduced by the porosity factor $p$, described earlier, to account for the foil winding approximation. This equation indicates how deeply the currents are distributed in the layer as a function of frequency and, therefore, the magnetic field distribution by Ampere’s Law. If $\delta$ is much greater than the conductor width $b$, the current and the magnetic field distribution are uniform, as shown for dc in Fig. 2(a). At higher frequencies, $\delta$ decreases and the majority of the current is carried at the surfaces of each conducting layer. Note, however, that the total current carried by a layer is the same regardless of frequency.
V. TRANSFORMER EQUIVALENT CIRCUIT

To introduce the measurement technique it is convenient to use the T-model transformer equivalent circuit shown in Fig. 3. The transformer model of Fig. 3(a) is used in this paper, while the model of Fig. 3(b) is more traditional. In the model of Fig. 3(a), winding 1 is represented by a winding resistance $R_1$ and an $N_1 : 1$ ideal transformer. Similarly, winding 2 is represented by $R_2$ and an $N_2 : 1$ ideal transformer. The elements $P_L$ and $P_M$ represent the permeance of the leakage flux paths for windings 1 and 2, respectively. The leakage flux is generated by $H_y$ which cuts through the electrically conductive but nonmagnetic transformer windings as illustrated in the prior section. The values of $P_L$ and $P_M$ are calculated from the leakage field and the leakage flux for the primary and secondary windings, respectively. The element $P_M$ models the permeance of the magnetizing flux path that links the two windings through the transformer core. In general, since $P_L$ and $P_M$ represent flux paths that traverse the nonmagnetic winding layers while $P_M$ represents flux paths in the high-permeability magnetic core, the values of $P_L$ and $P_M$ are much smaller than the value of $P_M$.

As illustrated by the FEA, and described subsequently in greater detail, the values of $R_1$, $R_2$, $P_L$, and $P_M$ are constant for low frequencies. With increasing frequency, the leakage flux generates eddy currents in the winding conductors. In Fig. 2(b), the eddy currents generated in the windings cause the change in the current distribution $J_z$ between dc and 8 kHz. The winding eddy currents circulate in a direction to oppose the leakage flux that creates them. The eddy currents increase transformer loss and, by opposing leakage flux, decrease the leakage flux. In Fig. 2(a), note that, in comparison to $H_y$ for dc which can be found by (10), $H_y$ for 8 kHz is reduced within the winding layers, but unchanged in the spaces between the windings. In the model, the increased loss is accounted for by increasing the effective ac values of $R_1$ and $R_2$ by some amount above the dc values. Similarly, the decrease in leakage flux is accounted for by decreasing the effective ac values of $P_L$ and $P_M$ as frequency increases.

The model of Fig. 3(a) is equivalent to the more traditional model shown in Fig. 3(b). In the traditional model, $P_L$ is referred through the $N_1 : 1$ ideal transformer to become the leakage inductance of winding 1 with value $L_1 = N_1^2 P_L$. Similarly, $P_M$ is referred through the $1 : N_2$ ideal transformer to become the leakage inductance of winding 2 with value $L_2 = N_2^2 P_M$. Finally, $P_M$ is referred to winding 1 to give a value of magnetizing inductance of $L_M = N_1^2 P_M$. Alternately $P_M$ could be referred to winding 2 to give a value of magnetizing inductance $L_M = N_2^2 P_M$. Therefore, when reporting a value for $L_M$ in the traditional model, the winding to which it has been referred must always be specified. To avoid this possible source of confusion, and for other reasons outlined subsequently, the model of Fig. 2(a) is preferred for the work reported in this paper. Note that the units of both permeance in Fig. 2(a) and inductance in Fig. 2(b) are expressed as Henries (H), since the number of turns in a winding is dimensionless. However, for dimensional analysis, the number of turns in a winding is often given the dimension “$t$” and the units of permeance are often expressed as “$H/t^2$” to distinguish a permeance from an inductance.

VI. MEASUREMENT INSTRUMENTATION

To characterize transformer effective ac winding resistance as a function of frequency, instrumentation for measurement of line impedance [2]–[4] is extended to wide-band measurement of transformer winding resistance and leakage permeance, as shown in Fig. 4. The transformer model of Fig. 3(a) is repeated in Fig. 4. The line model consists of a fundamental-frequency line-voltage $V_{L1}$, harmonics of the fundamental $V_{H1}$, and a line impedance $Z_L$. The instrumentation of Fig. 4 uses a network...
analyzer to accurately measure transformer effective ac resistance whether the transformer is energized with switch $S_1$ in position “1” or deenergized using a more conventional short-circuit measurement with $S_1$ in position “2.” An energized transformer can be measured while supplying a load with $S_2$ closed or in the no-load condition with switch $S_2$ open. Therefore, uniquely, this measurement method can be used to derate any transformer, regardless of whether it is energized, regardless of how it is loaded, and regardless of whether the dc winding resistance and line-frequency eddy-current loss is available for use in the calculation method of C57.110. This derating method is particularly valuable for older installed transformers that cannot be taken out of service and for which insufficient information prevents the use of C57.110.

In the measurement of Fig. 4, consider initially operation when switch $S_1$ is in position “1” and the transformer $N_1$-turn primary winding is connected to the line. Consider additionally that switch $S_2$ is closed and the $N_2$-turn secondary winding supplies current to the load. The local oscillator of the network analyzer generates a sinusoidal voltage at frequency $f_1$, which a power amplifier, $L$–$C$ filter, and isolation transformer inject as a small excitation current in parallel with the load current. The $L$–$C$ filter is tuned to line frequency to protect the amplifier from the large line-voltage fundamental appearing across the $N_2$-turn secondary winding while the isolation transformer permits nonground-referenced measurements. For the measurements reported in this paper, $f_1$ ranges from 10 Hz to 40 kHz. The current injected at $f_1$ is 40–60 dB smaller than the load current and does not disrupt transformer operation. It follows that the voltampere rating of the power amplifier is two or three orders of magnitude smaller than the transformer rating, and comparatively large transformers can be tested with small amplifiers.

A wide-band current sensor monitors the transformer current $I_T$ and a voltage sensor monitors the transformer voltage $V_T$. For symmetry, the voltage sensor is actually a second current sensor measuring the current flowing through a resistor connected across the $N_2$-turn transformer secondary winding. Note that, while some current injected at $f_1$ flows to the load, only the current actually flowing through the $N_2$-turn transformer secondary winding is measured by the current sensor and the load impedance does not affect the measurement.

The transformer voltage $V_T$ and current $I_T$ are supplied to the two input channels of the network analyzer and its two narrow-band filters centered at frequency $f_1$. Filter A finds the magnitude $|I_T|$ and phase $\angle I_T$ of the component of $I_T$ at $f_1$, while rejecting components present at the line frequency $f_L$ and the harmonic frequencies $f_H$. Similarly, filter B finds the magnitude $|V_T|$ and phase $\angle V_T$ of the component of $V_T$ at $f_1$ while rejecting components at $f_L$ and $f_H$. The network analyzer further calculates the apparent Thevenin impedance $Z_T$ at $f_1$ seen at the transformer secondary as

$$|Z_T| = \frac{|V_T|}{|I_T|},$$

$$\angle Z_T = \angle V_T - \angle I_T.$$  

Note that, with switch $S_1$ in position “1,” the instrumentation of Fig. 2 measures the transformer impedance in series with the line impedance $Z_L$. If the line impedance is significant, it is measured as described in [2]–[4] and subtracted from $Z_T$ to yield the true transformer impedance. The measured impedance $Z_T$, corrected for $Z_L$ if necessary, is equivalently represented as resistance $R_T$ and inductance $L_T$ using

$$R_T = |Z_T| \cos \angle Z_T,$$

$$L_T = \frac{1}{2\pi f_1 |Z_T|} \sin \angle Z_T.$$  

In terms of the circuit parameters in Fig. 3(a), the measurement gives

$$R_T = R_2 + \left(\frac{N_2}{N_1}\right)^2 R_1,$$

$$L_T = \frac{N_2^2 (P_1 + P_2)}{L_0}.$$  

The network analyzer repeats the measurement over a band of $f_1$ to characterize resistance and inductance as a function of frequency. Note that, at harmonic frequencies, the measured value of $R_T$ is equivalent to $R_0$ used in (8). The measurement of $L_T$ assumes that the impedance of the shunt magnetizing permeance $P_M$ is sufficiently large in comparison to the impedance of the leakage permeances $P_1$ and $P_2$, so that for most frequencies, the impedance of $P_M$ is much larger than the impedance of $P_1$ and $P_2$. However, as shown by the subsequent measurements, this assumption is violated for low frequencies, leading to a divergence between the measured and calculated values of $L_T$.

With switch $S_1$ in position “2,” the line is replaced with a short circuit and the values of $R_T$ and $L_T$ are measured directly without needing to correct for the line impedance $Z_L$. For this case, the measurement is essentially a wide-band short-circuit test.

### VII. THEORETICAL ANALYSIS

A theoretical analysis of transformer winding loss due to high-frequency currents has long been available and applied most extensively to small high-frequency transformers used in dc-to-dc converters. The theoretical analysis finds both the effective ac resistance and effective ac permeance as a function of frequency. Dowell, in 1966, was among the first to apply the equations to transformers and calculate effective ac resistance [6]. Since then, many authors have taken the same basic equations and applied them to specific situations. Reference [7] provides a good survey of efforts in this area. This theory has been successfully matched with short-circuit measurements of small deenergized, high-frequency transformers in [8]. This test is identical to the deenergized short-circuit measurement described previously. The theory presented in this discussion follows primarily from [8], which uses the notation published in [6].

The formulas for eddy-current loss are the solutions to differential equations for power loss in the winding layers. For the derivation, each layer in Fig. 1 is modeled as extending infinitely in the $z$ direction. However, in the final calculation, the length of each layer $l_{Tn}$ must be known to find the total volume of copper in the winding layer. The value of $H_y$ in
the spaces between the winding layers does not depend on frequency and defines the boundary condition for the surface of each layer as easily calculated from (10). Therefore, each layer has two boundary conditions, one at each surface. The resulting equations for resistance are listed below and the inductance equations are similar. For layer \( n \), the power loss density \( Q_n \) (W/m²) is found from (18), where for convenience, \( G_1 \) and \( G_2 \) are defined by (19) and (20).

\[
G_1(\Delta) = \Delta \frac{\sinh(2\Delta) + \sin(2\Delta)}{\cosh(2\Delta) - \cos(2\Delta)}
\]

\[
G_2(\Delta) = \Delta \frac{\sinh(\Delta) \cos(\Delta) + \cosh(\Delta) \sin(\Delta)}{\cosh(2\Delta) - \cos(2\Delta)}
\]

\[
Q_n = |H(x = b)|^2 [(1 + \alpha^2 + \beta^2)G_1(\Delta) - 4\alpha G_2(\Delta)].
\]

The quantity \( \Delta = b/\delta \) is the conductor thickness normalized to skin depth and \( (\alpha + j\beta) \) is a complex boundary condition ratio which simplifies the expressions. Reference [8] provides a complete description of these variables and boundary conditions. The total winding loss \( P_T \) is the power loss density \( Q_n \) for each layer multiplied by the area \( b_{\text{wim}}l_{Tn} \) of the layer summed over all \( N \) layers

\[
P_T = b_{\text{wim}} \sum_{n=1}^{N} l_{Tn} Q_n.
\]

The effective winding resistance \( R_T \) is the total power \( P_T \) divided by the square of the terminal current \( I \)

\[
R_T = \frac{P_T}{|I|^2}.
\]

The calculated value \( R_T \) in (22) corresponds to the measured \( R_T \) in (14) and, for the harmonic frequencies, \( R_0 \) defined in (8). A similar analysis finds the frequency dependence of the leakage inductance \( L_T \) as defined in (15).

VIII. TRANSFORMER MEASUREMENTS

Fig. 5 shows transformer resistance \( R_T \) and inductance \( L_T \) measured and calculated for the small test transformer shown in Fig. 1. The energized measurement with switch \( S_1 \) in position “1” closely matches the deenergized measurement with switch \( S_1 \) in position “2.” The energized measurement was performed with no load to avoid heating the transformer and changing the resistance of the winding in comparison to the deenergized measurement. Both measurements closely match the values calculated using the theoretical analysis and FEA. Note that, as described in [2]–[4], the measurement of the energized transformer with switch \( S_1 \) in position “1” suffers from interference near the fundamental frequency (60 Hz) caused by the line voltage fundamental \( V_L \) and near the odd harmonic frequencies (180 Hz, 300 Hz, 420 Hz, etc.) caused by the line voltage harmonics \( V_{Hh} \). However, the values of \( R_T \) and \( L_T \) are easily interpolated from measurements near the fundamental and harmonic frequencies. For example, the value of \( R_T \) cannot be measured for 60 Hz, but can be interpolated from the values of \( R_T \) measured at 50 and 70 Hz. Note also that the value of \( L_T \) rises for low frequencies in both measurements, but the value calculated from the theoretical and finite-element analyses does not rise. This error occurs because the measurement includes magnetizing permeance \( \mu_M \), the impedance of which is low for low frequencies, while the analyses assume that \( \mu_M \) is near infinite for all frequencies.

Fig. 6 shows transformer resistance \( R_T \) for the three distribution transformers described in Table II. Only the more convenient deenergized short-circuit measurements were performed. The winding designs are unknown and \( R_T \) cannot be found using either the theoretical or finite-element analyses. This is a strength of this measurement technique, since the winding design need not be known to obtain accurate data on transformer harmonic performance. Furthermore, this measurement technique provides accurate data for transformers with winding arrangements that do not conform to the assumptions necessary for theoretical analysis. The peaks in the 100-kVA resistance curve near 3 kHz indicate resonant points which are characteristic of the design. The general shape of the \( R_T \)-versus-frequency characteristics in Fig. 6 is similar to those for the test transformer of Fig. 5. However, the point at which \( R_T \) begins increasing with frequency changes dramatically depending on the transformer.
Fig. 6. Measured resistance for power system transformers listed in Table II.

<table>
<thead>
<tr>
<th>Rating</th>
<th>10 kVA</th>
<th>50 kVA</th>
<th>100 kVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>7.2 kV</td>
<td>200 V</td>
<td>250 kV</td>
</tr>
<tr>
<td>Secondary</td>
<td>240/120</td>
<td>200 V</td>
<td>480 V</td>
</tr>
</tbody>
</table>

IX. TRANSFORMER DERATING

Fig. 7 shows a portion of Fig. 6, but with linear axes and the frequency range restricted to 1800 Hz. Table III shows values of $R_h$ obtained from Fig. 7 for the first 19 odd harmonics. The data of Table III are used to derate the transformers for supplying harmonic currents. In normal operation, transformer windings are allowed to dissipate maximum load loss power $P_{LL,(max)}$ for the rated fundamental-frequency current $I_1$ and resistance $R_1$. For this case, (9) reduces to

$$P_{LL,(max)} = I_1^2 R_1$$

because there are no harmonic currents present. When harmonic currents $I_h$ are present, the load losses are the sum of the losses at rated current and resistance and the harmonic current and resistance as in (9). These total losses must remain less than $P_{LL,(max)}$, so the maximum fundamental frequency current $I_1$ must be reduced to prevent overheating. The value of $R_h$ increases with frequency due to eddy currents, so a constant resistance value is not sufficient. However, data obtained from the measurement listed in Table III provide these resistance values directly and precisely. This allows for accurate transformer derating for any given harmonic current spectrum.

In contrast, C57.110 assumes that the total resistance is composed of a dc resistance, an eddy-current resistance, and another stray loss resistance as shown in (8). The curves of Figs. 6 and 7 clearly do not fit (8), even over a moderate range of frequencies. Notice that the curve for the 50-kVA transformer in Fig. 6 is very similar in shape to the curve for the test transformer shown in Fig. 5(a). This is the characteristic shape of transformer ac resistance which is predicted by the theory. It shows three distinct regions. The first is a region of constant resistance where eddy-current effects are not important. This is followed by a region where the eddy-current resistance does increase quadratically with harmonic number. The final region shows the resistance increasing, but at a slower rate which is no longer quadratic. The other curves presumably conform to this general characteristic, but are shifted to higher or lower frequencies depending on the winding design. The resistance characteristic for the 100-kVA transformer begins to increase quadratically at low frequencies, while the resistance characteristic for the 10-kVA transformer remains nearly constant until 1 kHz.

X. CONCLUSION

This paper has presented a technique for direct measurement of transformer effective resistance as a function of frequency. The measurement is performed both on deenergized transformers and on energized transformers in either the no-load or loaded condition. The measured resistance-versus-frequency curves correspond well to an existing theoretical analysis and to an FEA. The resistance characteristics of several transformers of different voltampere ratings were measured and shown to differ dramatically. The results indicate that eddy-current losses are significant at different frequency ranges, so that a single derating technique may not be adequate. Finally, a derating method was introduced which uses the measured resistance at each harmonic frequency to provide more accurate transformer derating.

This measurement must be performed only once to characterize a transformer for any harmonic application. For new
transformers, it would seem to be particularly valuable to tabulate the effective ac resistance for harmonic frequencies, as shown in Table III. Given this resistance-versus-frequency characteristic, the transformer loss in a given application is calculated using the harmonic spectrum of the current to which the transformer is subjected. The accuracy and convenience of this combined measurement and calculation technique will help avoid failure of equipment improperly applied in a harmonic environment.

REFERENCES


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